Assignment 4.

1. Solve the equation $\sin(x - 30°) = 3\cos(x - 60°)$ for $-180° \leq x \leq 180°$. [5]

2. (a) Prove the identity $\cos\left(x + \frac{1}{6}\pi\right) + \sin\left(x + \frac{1}{3}\pi\right) \equiv \sqrt{3}\cos x$. [3]

   (b) Hence solve the equation $\cos\left(x + \frac{1}{6}\pi\right) + \sin\left(x + \frac{1}{3}\pi\right) = 1$ for $0 < x < 2\pi$. [3]

3. Solve the equation $\sec x = 4 - 2\tan^2 x$, giving all solutions in the interval $0° \leq x \leq 360°$. [6]

4. (a) Express $12\cos\theta - 5\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

   (b) Hence solve the equation $12\cos\theta - 5\sin\theta = 10$, giving all solutions in the interval $0° \leq \theta \leq 360°$. [4]

5. (a) Prove the identity $\tan\left(x+\frac{1}{4}\pi\right)+\tan\left(x-\frac{1}{4}\pi\right)\equiv 2\tan 2x$. [4]

(b) Hence solve the equation $\tan\left(x+\frac{1}{4}\pi\right)+\tan\left(x-\frac{1}{4}\pi\right)=2$, for $0\le x\le\pi$. [3]

Total mark of this assignment: 31.

(†) Bonus questions:

1. Show that $\tan 3\theta = \dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$.
   Given that $\theta=\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and that θ is acute, show that $\tan 3\theta=\frac{11}{2}$.
   Hence find all solutions of the equations.
   (a) $\tan\left(3\cos^{-1}x\right)=\frac{11}{2}$,
   (b) $\cos\left(\frac{1}{3}\tan^{-1}y\right)=\frac{2}{\sqrt{5}}$.

2. The sides of a triangle have lengths $x-y$, x and $x+y$, where $x>y>0$. The largest and smallest angles of the triangle are α and β, respectively. Show that

$$4(1-\cos\alpha)(1-\cos\beta)=\cos\alpha+\cos\beta.$$

   In the case $\alpha=2\beta$, show that $\cos\beta=\frac{3}{4}$ and hence find the ratio of the lengths of the sides of the triangle.

Take this part back home and work on these for fun!